Maximum likelihood estimation in Gaussian distributions under total positivity

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The problem of maximum likelihood estimation for Gaussian distributions that are multivariate totally positive of order two (MTP2) is investigated. The maximum likelihood estimator (MLE) for such distributions exists based on just two observations, irrespective of the underlying dimension. It is further demonstrated that the MTP2 constraint serves as an implicit regularizer and leads to sparsity in the estimated inverse covariance matrix, determining what we name the ML graph. It is shown that the maximum weight spanning forest (MWSF) of the empirical correlation matrix is a spanning forest of the ML graph. In addition, an upper bound for the ML graph is found by adding edges to the MSWF corresponding to correlations in excess of those explained by the forest. Globally convergent coordinate descent algorithms for calculating the MLE under the MTP2 constraint are provided, structurally similar to iterative proportional scaling. The lecture is based on joint work with Caroline Uhler and Piotr Zwiernik.

References

[1] Lauritzen, S., Uhler, C., and Zwiernik, P. (2017). Maximum likelihood estimation in Gaussian models under total positivity. To appear in *The Annals of Statistics*. arXiv:1702.04031.